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FUZZY PREFERENCE METHOD FOR DEFINING GROUP PREFERENCES.(U)

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THESIS

FUZZY PREFERENCE METHOD FOR DEFINING GROUP PREFERENCES

by

Robert David Clarke

March 1980

Thesis Advisor: Donald R. Barr

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This paper presents a 'fuzzy preference' method of defining 'group preference' among alternatives based on the individuals' preferences. Group strengths of preference are also defined on the basis of individual strengths. An example and the results of four experiments are presented for the fuzzy preference method. Brief discussions are given of definitions and extensions in fuzzy set theory.

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Fuzzy Preference Method for Defining Group Preferences

by

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Major, United States Marine Corps
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Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

From the

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March 1980

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ABSTRACT

This paper presents a 'fuzzy preference' method of defining 'group preference' among alternatives based on the individuals' preferences. Group strengths of preference are also defined on the basis of individual strengths. An example and the results of four experiments are presented for the fuzzy preference method. Brief discussions are given of definitions and extensions in fuzzy set theory.

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I. INTRODUCTION

The goal of this writer is to present a simple, useable method of assisting decision-makers in solving a particular type of problem. The problem is how to use individual preferences to form the overall preferences of a 'group' of individuals. The reader having a modest background in mathematics should find this paper an interesting treatment of this type of problem. It is a problem often encountered in eliciting the group preference from a team of advisors. What are some of the difficulties faced by decision-makers in defining the preferences of a group of individuals over a set of alternatives?

The social, political, business, and military atmosphere today is clouded by a mood of complexity, ambiquity, and uncertainty. Increasingly more variables enter the arena; values change across the scope of time and personalities; the future continues to be quite unpredictable. Decision-makers are challenged to achieve more production with fewer expenditures. Commanders, managers, and analysts are frustrated by their inability to adequately capture the essence of their problems and to consistently render satisfactory solutions.

Zadeh [1] stated the 'principle of incompatibility' which gives us a sense of direction when we are confronted with the complexity of modern decision making.

> "The closer one looks at a 'real world' problem, the fuzzier becomes its solution. Stated informally, the essence of this principle is that as the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics."

According to Zadeh, fuzziness, vagueness, and imprecision are terms with shady connotations. Precision, logic, and clarity are terms perpetuated by the rigors of mathematics. "As we learn more about human cognition, we may well arrive at the realization that man's ability to manipulate fuzzy concepts is a major asset rather than a liability, and it is this ability, above all, that constitutes a key to the understanding of the profound difference between human intelligence, on one hand, and machine intelligence, on the other." [2] Therein lies the motivation for the language and logic of fuzzy sets.

Decision-making is an art and science of considerable scope. Extensive work has been done in many aspects of the theory and application of decision-making. The aim of this paper is to present just one of these aspects - preference ordering of alternatives for a group of individuals. Decision opportunities carry the goal of selecting from among alternatives the best choice or most favorable alternative. Implicit in the analytical process of making a choice is a preference ordering of alternatives. At least, a decision maker chooses one (or more) alternatives as best and the remainder as second

best. At most, the decision maker assigns a complete ordering of the alternatives according to his preferences. This process is inherently difficult for one person to do; for a group of people to arrive at a preferential concensus over a list of alternatives can be an arduous task. This paper presents a method for establishing a complete ordering of the alternatives for the group, based on the preferences of individual group members.

Section two of this paper presents some basic definitions of the theory for dealing with imprecision. Section three decribes an application of fuzzy set theory, group preference ordering based on individual preferences. In section four there is a brief overview of literature and other applications of fuzzy set theory.

II. FUZZY SET THEORY

Fuzzy set theory was introduced by Zadeh for handling vague, inexact information in a mathematically rigorous way. In ordinary or 'crisp' set theory, each element of a particular set's universe of discourse either belongs or doesn't belong to the particular set. The universe of discourse is the set of all objects, defined by enumeration or rule, that will be considered in a given context. If, for instance, the universe of discourse is the set of positive integers, the set A of odd numbers less that ten is $A = \{1,3,5,7,9\}$. The set A is well defined, i.e., given any object it can be determined whether the object is in the set.

A fuzzy set taken over the same universe of discourse is the set B of odd numbers 'close to ten'. The characteristic function of a fuzzy set is allowed to take any value between 0 and 1 inclusive. Therefore, all elements in the universe of discourse 'belong' to a given fuzzy set, but with possibly different grades of membership. The membership function of the fuzzy set B is denoted by $\mu_B(x)$, where x varies over the universal set χ. Our set B might, for example, have the following membership function values:

$$\mu_B(9) = \mu_B(11) = 1.0$$

$$\mu_B(7) = \mu_B(13) = .75$$

$$\mu_B(5) = \mu_B(15) = .5$$

$$\mu_B(3) = \mu_B(17) = .25$$

$$\mu_B(1) = \mu_B(19) = \mu_B(21) = \ldots = 0$$

The membership function for the fuzzy set B is represented graphically in Figure 2.1.

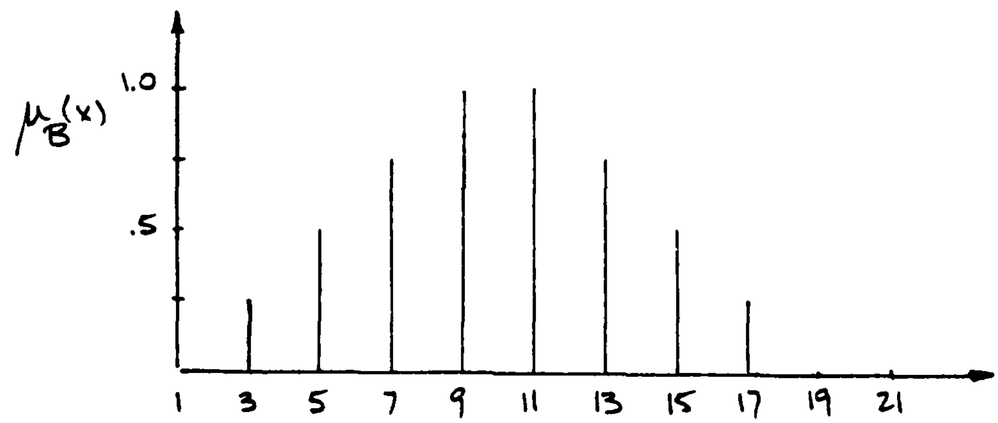


Figure 2.1 - MEMBERSHIP FUNCTION FOR THE FUZZY SET B

Fuzzy set theory is an extension of ordinary set theory. Basic definitions and relations to fuzzy set theory, such as union, intersection, and complementation, are extensions of the corresponding definitions in ordinary set theory.

The union of two fuzzy sets A and B on χ, denoted $A \cup B$, a fuzzy set, is defined by the membership function

$$\mu_{A \cup B}(x) = \max\{\mu_A(x); \mu_B(x)\}.$$

The intersection of two fuzzy sets A and B on χ, denoted $A \cap B$, is a fuzzy set defined by the membership function

$$\mu_{A \cap B}(x) = \min\{\mu_A(x); \mu_B(x)\}.$$

The complement of A, denoted $\bar{A}$, is a fuzzy set defined by

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x).$$

Many algebraic properties from ordinary set theory also hold: commutativity, associativity, distributivity, and DeMorgan's theorems. Linguistic hedges such as 'very' and 'somewhat' operate on fuzzy membership functions to change their 'meanings'. Suppose, for example, the fuzzy set $A = \{young\}$ were defined by the membership function

$$\mu_A(x) = (1 + (.04x)^2)^{-1}$$

where the universe of discourse is ages of people, in a continuous sense. The membership function for the fuzzy set $B = \{very\ young\}$ might, for example, be given as the square of the former function.

$$\mu_B(x) = (1 + (.04x)^2)^{-2}.$$

On the other hand, suppose we define the membership function for fuzzy set $C = \{somewhat\ young\}$ as

$$\mu_C(x) = (1 + (.04x)^2)^{-1/2}.$$

The membership functions for these three contrived sets are graphically represented in Figure 2.2.

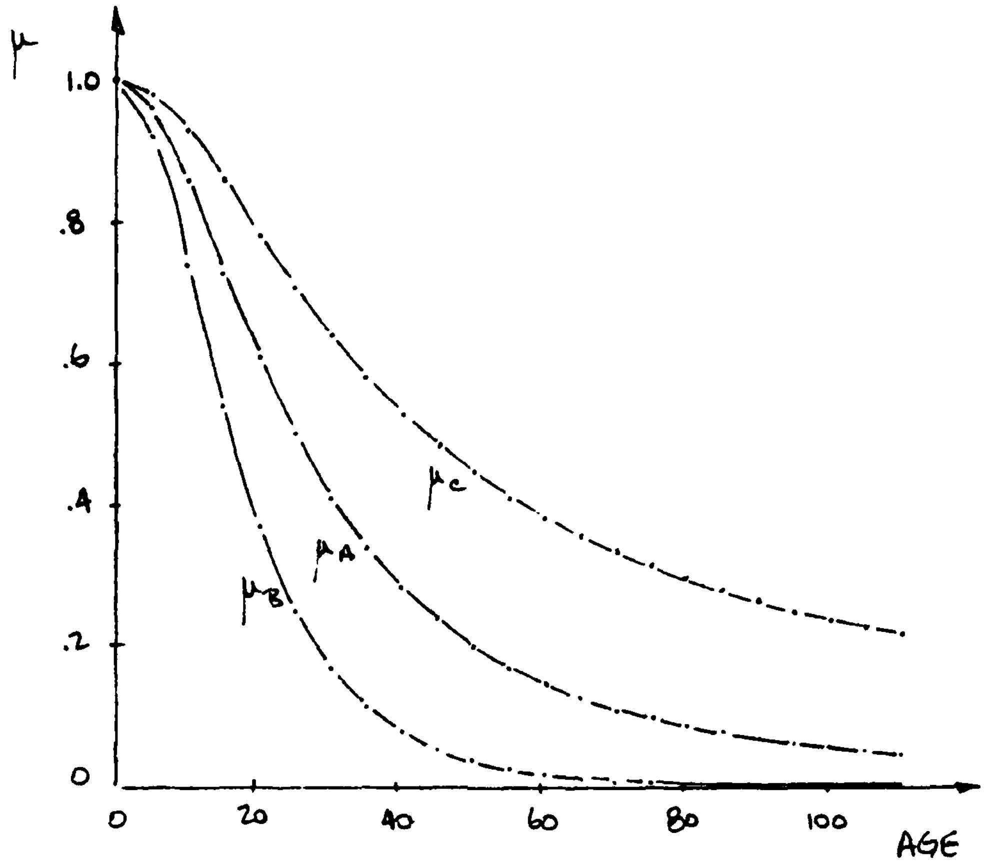


Figure 2.2 - MEMBERSHIP FUNCTIONS FOR THE FUZZY SETS A, B, C

The α-level set of a fuzzy set D on χ is defined as the ordinary (non-fuzzy) set $S_\alpha(D)$ for which the degree of membership in D exceeds or equals the level α :

$$S_\alpha(D) = \{x \mid \mu_D(x) \geq \alpha\}.$$

This is illustrated in Figure 2.3.

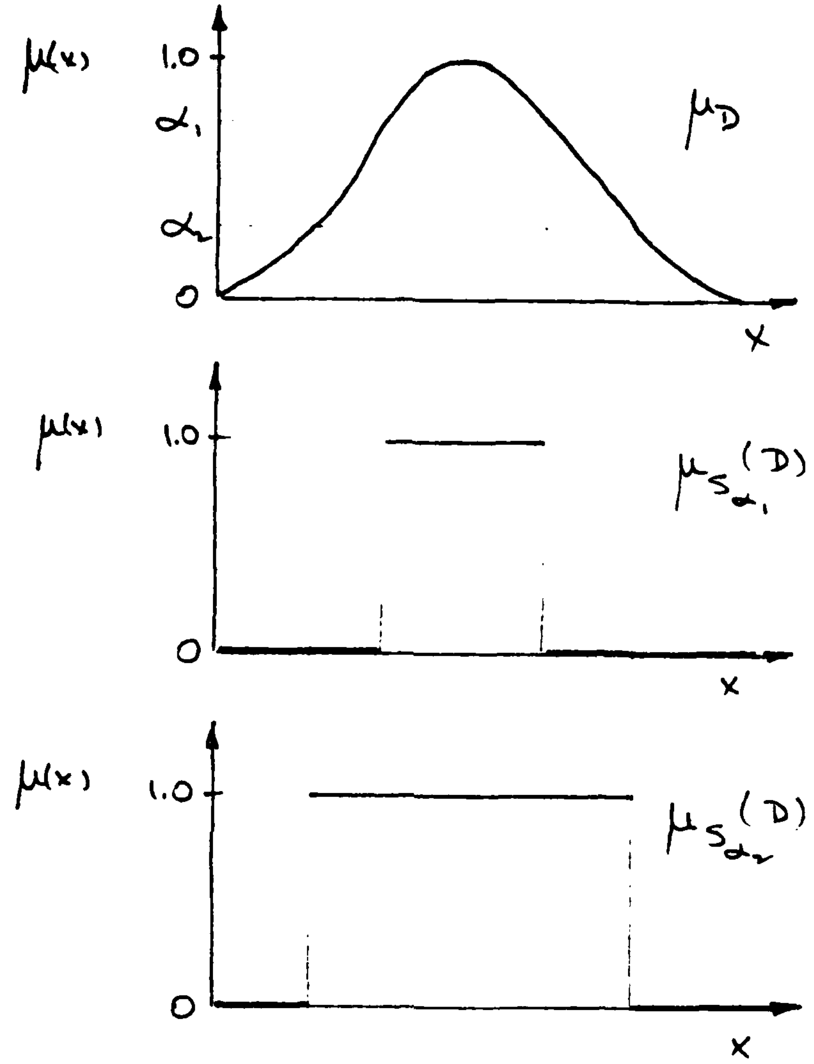


Figure 2.3 - α-LEVEL SETS OF A FUZZY SET D

A simple example follows that demonstrates an application of fuzzy sets in finding a fuzzy solution to a pair of competing objectives. In the example the objectives, expressed as fuzzy sets are

A: x should be much larger than 5

B: x should be near to 10

where:

$$\mu_A(x) = 1 - (1 + (0.2\ (x - 5))^2)^{-1} \quad \text{for } x > 5$$

$$= 0 \quad \text{for } x \leq 5$$

$$\mu_B(x) = (1 + (x - 10)^2)^{-1}$$

A question that might be asked is what values of x have positive membership in a fuzzy set that is much larger than 5 and near 10 . What is the value for x that gives maximum membership value in this intersection? The intersection of the two fuzzy sets is the fuzzy set C = {much larger than 5 and near 10} , has the following membership function:

$$\mu_C(x) = \min\{\mu_A(x);\ \mu_B(x)\}$$

and is illustrated in Figure 2.4. All of the values of x greater than 5 have positive membership value. It can be seen on the membership function for C that all values of x greater than 5 have positive membership value. It can also be seen that x equal 10.85 has the maximum membership value. This example briefly shows one useful aspect of fuzzy set theory.

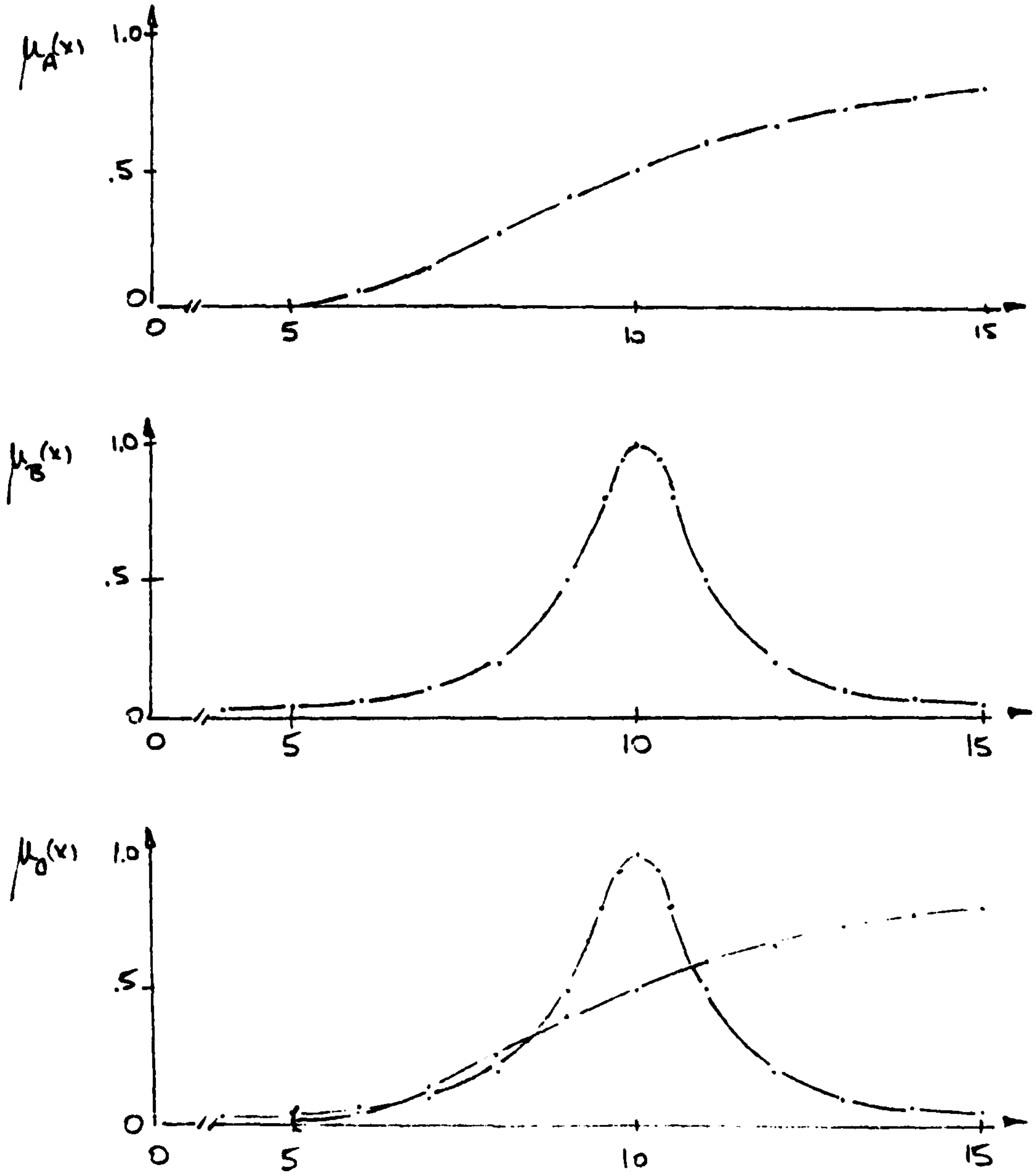


Figure 2.4 - MEMBERSHIP FUNCTIONS FOR FUZZY SETS A, B, C

This concludes a very brief overview of some definitions for fuzzy set theory. Fuzzy set theory is useful in defining group preferences based on individual preferences. This is discussed in section three. Extensions and a short literature review for fuzzy set theory are included in section four.

III. FUZZY PREFERENCE METHOD

Imprecision seems to be a characteristic of 'real world' problems. Fuzzy set theory appears to provide models useful in solving imprecise problems in a simple and useable manner. In this section we will outline a method of defining group preferences, based on the preferences of individuals within the group.

Group preference ordering is the collective ranking of alternatives, based on the preferences of individuals within the group. This paper describes only one of many ways to define group preferences. We assume that each group member can rank the alternatives according to his personal preferences. The problem at hand is to find an ordering of the alternatives consistent, in some sense, with the preferences of the 'group.' This problem occurs frequently in military, business, and social environments. Hierarchical structures lend themselves to staffs and committees which are tasked to present their preferences over the considered alternatives. What is a simple, useable method to define the preferences of a group?

Blin and Whinston [3] describe an appealing method for establishing and presenting the ranking of alternatives for a group, based on the rankings of individuals within the group. Generally, Blin and Whinston constructed a preference matrix A, composed of elements a_{ij} . Each element indicated the preference for the individual of alternative i over alternative j . In the Blin and Whinston model elements a_{ij}

were either 1 (alternative i is preferred over j) or 0 (alternative i is not preferred over j .) To establish preferences for the group, individual matrices were summed together to form an aggregation matrix B .

$$B = \sum_{k=1}^{M} A_k \qquad \text{For } M = \text{number of individuals}$$

Blin and Whinston divided the aggregation matrix B by the number of individuals M to yield the group preference matrix C .

$$C = \frac{B}{M}$$

The elements C_{ij} of the group preference matrix C defined the group preference for the alternative i over alternative j . From the group preference matrix α-level sets were constructed to display 'agreement levels' for the group.

The fuzzy preference model presented in this paper is a variation of the Blin and Whinston method. The first difference in this model occurs in the construction of the individual group member preference matrix A . The elements of this matrix (a_{ij}) can assume values from 0 to 1.0 inclusive. This variation allows strengths of preference to be incorporated into the model. Strength of preference is a subjective measure of personal determination regarding the choice of one alternative over another. Strength of preference equal to one shows full resolve in the preference of one alternative

over another. Strength equal to zero indicates indifference in preference. The second difference in this paper is the weighting of individual preference matrices. Weighting allows the preferences and strengths of selected individual to have greater impact on the group preference ordering. In the weighted case the aggregation matrix is

$$B = \sum_{k=1}^{M} W_k A_k$$

where the W_k are weights for each individual. The group preference matrix becomes:

$$C = \frac{B}{\sum_{k=1}^{M} W_k} .$$

Aside from these two differences, the methodology and interpretation of our method remains the same as the Blin and Whinston model. An example and the results of four experiments are presented to demonstrate the simplicity of this method and suggest its usefulness. Four alternatives are treated in the example and experiments; a larger number could be readily managed. In small dimensional problems, the calculations required for this method are simple enough to be carried out by hand.

The first step of our fuzzy preference method is obtaining the preference ordering of alternatives by each individual group member. Two types of information should be

collected in this step. The first is a ranking of alternatives from most to least preferred. In the case where an alternative is not ranked, every element a_{ij} in the individual preference matrix carrying the subscript of the unranked alternative will be set to zero (row and column). The second type of information is the strength of preference between every two consecutive alternatives in the ranked list. If no members of a group recorded strengths of preference, the values should be set equal to one, as in the Blin and Whinston model. If a small number of group members did not record strengths of preference, values for their individual preference matrices may be set to one half or another arbitrary 'averaging' value. An alternate solution is reversion to the Blin and Whinston model with unity strengths for all individual preferences. An individual preference matrix with strengths of preference partially missing should be treated similar to the previous situation - averaging or reversion to unity strengths for all individual preferences.

Suppose, for example, that a person were ranking four alternate activities for Saturday afternoon. Alternatives might be:

A. mow the lawn

B. play tennis

C. wash the car

D. swim at the pool

An outline for ranking these alternatives could be:

$$\quad > \quad > \quad > \quad$$

The symbol '>' is interpreted 'is preferred to' and separates the positions for entering alternatives. For example, the following:

$$B > \quad D > \quad A > \quad C$$

would be read; playing tennis is most preferred, followed by swimming, then mowing the lawn, and finally, washing the car. Continuing in the example, strengths of preference of an individual might be:

$$\begin{array}{ccccccc} B & > & D & > & A & > & C \\ & .5 & & 1.0 & & .2 & \end{array}$$

Strength of preference can be any value between zero (no preference) and one (strong preference.) In our example, B and D are strongly preferred over A and C . Tennis is preferred over swimming, but with medium strength. Lawn mowing is preferred over washing the car, but only slightly.

The second step of fuzzy preference method is plotting the individual preferences from step one in a simple, square matrix format. The matrix is as large as the square of the number of alternatives being considered. In our example of activity preferences, the matrix has four rows and four columns.

	A	B	C	D
A				
B			*	
C				
D				

Figure 3.1 - INDIVIDUAL PREFERENCE MATRIX

A single element a_{ij} of this preference matrix A is interpreted as the individual's strength of preference of that row (representative alternative) over the column (representative alternative.) For example, the asterisk '*' position will contain the individual's strength of preference for B over C . Even before our example values are filled in we see that each diagonal value is zero, since an alternative has zero preference over itself. With our example values, construction of the preference matrix begins as follows:

	A	B	C	D
A	0			
B	1	0	1	.5
C			0	
D				0

Figure 3.2 - INDIVIDUAL PREFERENCE MATRIX

Entry of preferences of B over the other alternatives shows; B is preferred over A with strength 1.0, B is preferred over C with strength 1.0 and B is preferred over D with a strength .5 . Completing the example individual preference matrix yields Figure 3.3.

	A	B	C	D
A	0	0	.2	0
B	1	0	1	.5
C	0	0	0	0
D	1	0	1	0

Figure 3.3 - COMPLETED INDIVIDUAL PREFERENCE MATRIX

Strength of preference is a successive maximizing operation for each alternative. In our example the strength of preference for B over D is .5 while the strength of preference for B over A is max {.5, 1.0} or 1.0. The strength of preference for B over C is max {.5, 1.0, .2} or 1.0. This maximizing operation is pertinent to every successive strength of preference relationship. Alternative C is not preferred over any alternative. No alternative is preferred over B. At a glance, all preferences and strengths are available from the individual preference matrix.

Step three in our fuzzy preference method concerns aggregation of individual preference matrices. The aggregation

matrix B is constructed by summing the individual preference matrices.

$$B = \sum_{k=1}^{M} A_k$$

In the continuation of our example, suppose that six members of a group participated in the Saturday activity preference ordering. The following aggregation matrix B is contrived for the purpose of our example.

	A	B	C	D
A	0	.5	2.0	.3
B	5.5	0	5.3	2.0
C	1.5	.2	0	.7
D	5.6	2.1	5.0	0

Figure 3.4 - AGGREGATION MATRIX

Step four concerns the normalization of the aggregation matrix. In the case of equal weights among individual preference matrices the aggregation matrix B is divided by the number of group members M (in this example M = 6) to yield the group preference C.

$$C = \frac{B}{M}$$

The case of unequal weights is discussed later in this section. Figure 3.5 is the group preference matrix for our example.

	A	B	C	D
A	0	.08	.33	.05
B	.92	0	.88	.33
C	.25	.03	0	.12
D	.93	.35	.83	0

Figure 3.5 - GROUP PREFERENCE MATRIX

Figure 3.5 displays information regarding preferences and strengths of preference as defined for the 'group.' Each element c_{ij} of the group preference matrix yields a sense of membership for alternative preference comparisons.

Step five of our method is to determine α-level sets for the group preference matrix C. Recalling from section two, an α-level set is the set of all elements having membership at least as large as the value of α. In the example the α-level set at $\alpha = 1.0$ is empty, as can be seen from Figure 3.5. At $\alpha = .93$ there is one preference, namely D is preferred over A (with strength of preference = .93 .) The α-level set for $\alpha = .93$ has one element, D preferred over A. This set is denoted as follows:

$$R_{.93} = \{(D,A)\}.$$

As you recall, the alternatives are:

A. mow the lawn

B. play tennis

C. wash the car

D. swim at the pool

Figure 3.6 should aid in understanding the preference ordering.

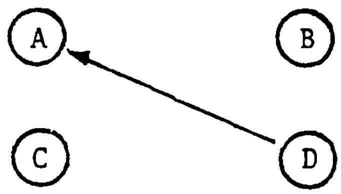


Figure 3.6 - DIAGRAM OF PREFERENCE ORDERING

At $\alpha = .92$ a second relationship occurs and the α-level set at $\alpha = .92$ has two members - (D,A) and (B,A).

$$R_{.92} = \{(D,A) \ , \ (B,A)\}$$

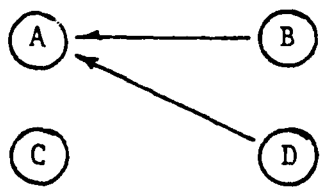
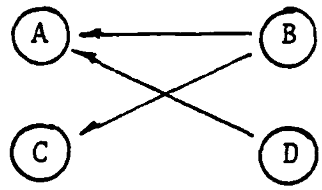


Figure 3.7 - DIAGRAM OF PREFERENCE ORDERING

Figure 3.7 indicates that alternative A is strongly not preferred. So far, we have not defined a complete ordering for every alternative for the group.

Subsequent α-level sets are constructed in like manner.

$$R_{.88} = \{(D,A),(B,A),(B,C)\}$$



$$R_{.83} = \{(D,A),(B,A),(B,C),(D,C)\}$$

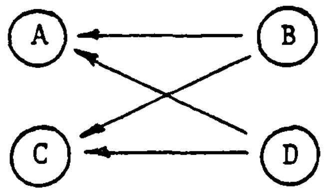
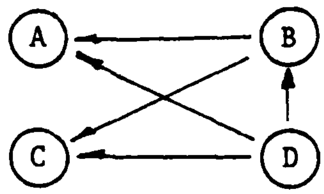


Figure 3.8 - DIAGRAMS OF PREFERENCE ORDERING

At this point we see that the group prefers B and D over A and C. There is yet to appear, in our α-level set constructions, a relationship of A with respect to C and B with respect to D.

$R_{.35} = \{(D,A),(B,A),(B,C),(D,C),(D,B)\}$



$R_{.33} = \{(D,A),(B,A),(B,C),(D,C),(D,B),(A,C)\}$

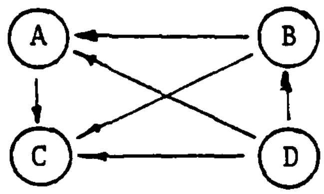


Figure 3.9 - DIAGRAMS OF PREFERENCE ORDERING

A complete ordering of the four alternatives is revealed in $R_{.33}$. For the example the fuzzy preference of the group is

D >	B >	A >	C
.35	.92	.33	

The values in the blocks are the strengths that accompany the fuzzy preferences. Strong preferences were defined for D and B over A and C, but weak preferences for D over B and A over C.

A variation to this method allows the decision-maker to assign weighting factors to those group members whose opinions are more valued by him for a particular set of alternatives. In our example, suppose that one member's preference matrix is weighted by a factor of two. In this sense, the particular member's preferences are twice as valuable to the

decision-maker as any other member. Recalling from before, the aggregation matrix for weighting is

$$B = \sum_{k=1}^{M} W_k A_k.$$

Figure 3.10 is a contrived aggregation matrix for the Saturday activity example, with double weighting for one group member.

	A	B	C	D
A	0	.5	2.4	.3
B	6.4	0	6.3	2.3
C	1.5	.2	0	.7
D	6.0	2.1	5.4	0

Figure 3.10 - AGGREGATION MATRIX WITH DOUBLE WEIGHTING

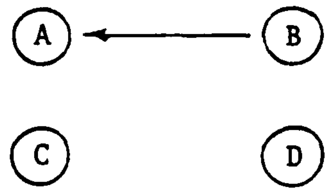
Step four normalizes the aggregation matrix B by dividing by the sum of the weights ($\sum_{k=1}^{M} W_k = 7$, in our double-weighted example.) The weighting factor gave the result as if a seventh group member were casting a preference vote equal to the 'more valuable' member preference. The group preference matrix with weighting is shown in Figure 3.11.

	A	B	C	D
A	0	.07	.34	.04
B	.91	.0	.90	.33
C	.21	.03	0	.10
D	.86	.30	.77	0

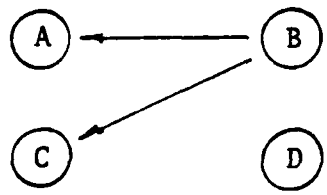
Figure 3.11 - GROUP PREFERENCE MATRIX WITH DOUBLE WEIGHTING

The new α-level sets are similar to the unweighted example with an important exception at $\alpha = .33$. Figure 3.12 shows the construction of the weighted α-level sets.

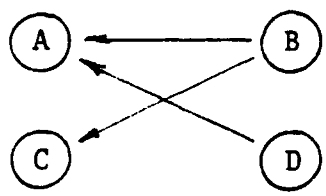
$R_{.91} = \{(B,A)\}$



$R_{.90} = \{(B,A),(B,C)\}$



$R_{.86} = \{(B,A),(B,C),(D,A)\}$



$R_{.77} = \{(B,A),(B,C),(D,A),(D,C)\}$

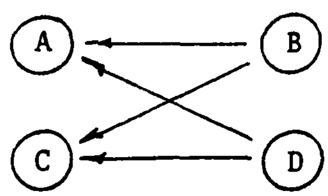


Figure 3.12 - DIAGRAMS OF PREFERENCE ORDERING WITH DOUBLE WEIGHTING

At this point we are quite close to the unweighted case where $\alpha = .83$.

$$R_{.34} = \{(B,A),(B,C),(D,A),(D,C),(A,C)\}$$

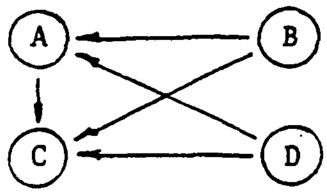


Figure 3.13 - DIAGRAM OF PREFERENCE ORDERING WITH DOUBLE WEIGHTING

At $\alpha = .33$ the double weighting of a member (who preferred B over D) causes a reversal in the group's fuzzy preference between B and D.

$$R_{.33} = \{(B,A),(B,C),(D,A),(D,C),(A,C),(B,D)\}$$

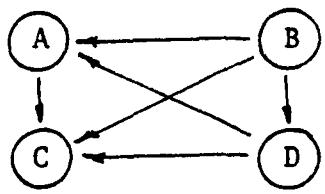


Figure 3.14 - DIAGRAM OF PREFERENCE ORDERING WITH DOUBLE WEIGHTING

$R_{.33}$ defines a complete ordering of the four alternatives.

$$\underset{.33}{B >} \quad \underset{.86}{D >} \quad \underset{.34}{A >} \quad C$$

The preference ordering of B and D has changed. The strengths of fuzzy preferences are shown in Figure 3.11. The preference

of B over D has strength of .33, while the preference of D over B has strength of .30 . B is only weakly preferred over D, but B and D are strongly preferred over A and C .

An outline of the fuzzy preference method is contained in Appendix A.

Experiments were conducted to test the simplicity and usefulness of the fuzzy preference method. Sets of alternatives were chosen so as to be familiar to the participants and to generate heterogeneous results. The fuzzy preference method was exercised in the unweighted mode by twelve volunteer graduate students at the Naval Postgraduate School. The results of four experiments, each with four alternatives, are shown in Figures 3.15 - 3.18. Each figure gives the alternatives, group preference matrix, α-level set at which complete ordering of alternatives is achieved, and the fuzzy preferences of alternatives.

The first experiment was a choice of foods:

Alternatives

A. Roast Leg of Lamb

B. Prime Rib of Beef

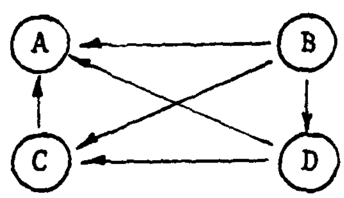
C. King Crab

D. Lobster

Group Preference Matrix

	A	B	C	D
A	0	.08	.18	.13
B	.64	0	.51	.43
C	.57	.26	0	.08
D	.66	.32	.46	0

$R_{.43} = \{(B,D),(B,C),(B,A),(D,C),(D,A),(C,A)\}$



B > D > C > A

.43 .46 .57

Figure 3.15 - RESULTS OF FOOD EXPERIMENT

The second experiment was a preference ordering of participant sports:

Alternatives

A. Water Skiing

B. Tennis

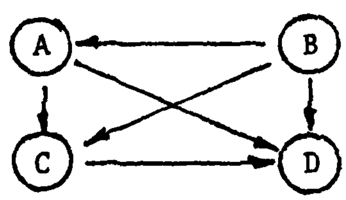
C. Bowling

D. Roller Skating

Group Preference Matrix

	A	B	C	D
A	0	.28	.41	.54
B	.39	0	.68	.63
C	.32	.18	0	.53
D	.19	.22	.06	0

$$R_{.39} = \{(B,A),(B,C),(B,D),(A,C),(A,D),(C,D)\}$$



B > A > C > D

.39 .41 .53

Figure 3.16 - RESULTS OF SPORTS EXPERIMENT

The third experiment was preference ordering of retirement location.

Alternatives

A. Northeast

B. Northwest

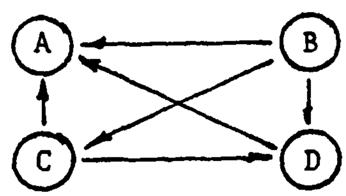
C. Southeast

D. Southwest

Group Preference Matrix

	A	B	C	D
A	0	.04	.15	.12
B	.73	0	.40	.39
C	.50	.29	0	.38
D	.64	.20	.30	0

$R_{.38} = \{(B,C),(B,D),(B,A),(C,D),(C,A),(D,A)\}$



B >	C >	D >	A
.40	.38	.64	

Figure 3.17 - RESULTS OF RETIREMENT EXPERIMENT

The fourth experiment was preference ordering of music:

Alternatives

A. Rock and Roll

B. Disco

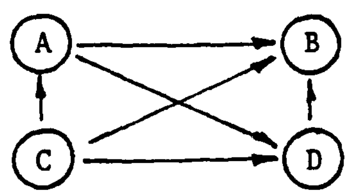
C. Classical

D. Jazz

Group Preference Matrix

	A	B	C	D
A	0	.25	.22	.35
B	.13	0	.12	.30
C	.48	.57	0	.69
D	.27	.34	.08	0

$R_{.34} = \{(C,A),(C,D),(C,B),(A,D),(A,B),(D,B)\}$



C > A > D > B

.48 .35 .34

Figure 3.18 - RESULTS OF MUSIC EXPERIMENT

The results of the example problem and four experiments reveal the simplicity of the fuzzy preference method. Usefulness of this method is suggested by these applications. Defining the preference of a group can be a valuable aid to decision-makers. Strength of preference helps to show the decision-maker the magnitude of group support for each preference.

In the concluding section of this paper we briefly discuss extensions and criticism of this model and introduce literature for further study.

IV. CONCLUSION

In concluding this presentation we look backward and forward. First, did we answer our initial questions? We have found an intuitively appealing aid for decision-makers to elicit group preferences based on individual preferences over a set of alternatives. It is a simple, useable model that shows order and strength of preferences for a group of individuals.

Critically, the greatest possibility of difficulty occurs in group members making their preferences and strengths of preferences. This process is highly dependent on subjective judgement and exhibits the strong and weak characteristics of subjectivity. Favorably, subjectivity allows a person to bring to bear on the problem quantitative and qualitative information, experience, and intuition. Unfavorably, subjectivity invites personal prejudice and unrelated bias to affect the outcome. Also, the number of alternatives should be small enough to allow group members to ascertain their preferences over the entire set of alternatives.

With this model, decision-makers have the option of biasing the group preferences by weighting the preferences of selected members of his group. In a problem, for instance, oriented towards operations a commander might weight the preferences of the operations staff officer higher than those of his other group members.

Looking ahead, our method can be extended in many directions. Different weighting schemes would give decision-makers flexibility in determing group preferences. Another approach is an extension that would allow group members to assess their own level of expertise. With a high self-assessment, a member would weigh his preference higher, compared to fellow members. With a low self-assessment, a member weights in a way to detract from his preference contribution to the group aggregation matrix.

Considerable work has been done in fuzzy set theory. Zadeh pioneered the study of fuzzy sets with a paper in 1965 [4]. Since then he has published papers on linguistic variables, approximate reasoning, and fuzzy languages [5]. Bellman and Zadeh published a paper in 1970 that provided a basis for multi-criteria decision making in fuzzy sets [6]. A more recent presentation on multicriteria decision making was published by Blin [7]. Fuzzy preference functions have been investigated by Roy [8] and Bezdek [9]. Mathematical programming, considering decision problems with special structure, has been presented by Tanaka, Okuda, Asai [10], and Zimmermann [11]. Yager [12] has considered general decision making in fuzzy environments.

Fuzzy set theory has found application in many areas besides decision theory. Fuzzy methodologies in matching have been applied to criminal investigation [13], personnel management [14], and information processing [15]. Artificial

intelligence [16] and fuzzy control [17,18] are areas explored in fuzzy linguistics. Medical diagnosis using fuzzy set theory has been done by Esogbue [19] and Wechsler [20]. Applications of fuzzy set theory have also been made to psychological problems [21]. A bibliography written by Gaines and Kohout [22] is an excellent guide through fuzzy set theory and it's applications.

APPENDIX A

OUTLINE OF THE FUZZY PREFERENCE PROCEDURE

Step 1 Each group member records his preferences and strength of preferences over the alternatives.

> > >

Step 2 Plot each record of preference in a square matrix format.

	A	B	C	D
A	0			
B		0		
C			0	
D				0

Step 3 Combine individual preference matrices with weights in one aggregation matrix.

$$B = \sum_{k=1}^{M} W_k A_k$$

Step 4 Normalize the aggregation matrix to create the group preference matrix

$$C = \frac{B}{\Sigma W_k}.$$

Step 5 Determine the α-level sets from the group preference matrix. Finally, indicate the complete ordering of the alternatives.

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